Solutions of final exam questions
Set $A = B = C$

True/False

(II) T  (III) T  (IV) T  (V) F  (VI) T
(VII) T  (VIII) T  (IX) F  (X) T
For steady, two-dimensional flow, the rate of change of temperature is

\[ \frac{dT}{dt} = u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = (x^2 y^2 + x) \frac{8x}{9} + (2xy - y)(-9y^2) \]

At \((x,y) = (2,1)\), \[ \frac{dT}{dt} = (5)(14) - 5(-9) = 125 \text{ units} \]
Incompressible continuity equation is given by

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial q}{\partial z} = 0 \]

\[ \Rightarrow \frac{\partial}{\partial x} (4xy^2) + \frac{\partial}{\partial y} (-2y^2) = 4y^2 + \frac{db}{dy} - y^2 = 0 \]

\[ \Rightarrow \frac{db}{dy} = -3y^2 \], integrate: \[ f(y) = \int (3y^2) \, dy = -y^3 + \text{constant} \]
The relevant dimensions are \( z_3 = \{ m, \ell^2 \}, \ \bar{z}_3 = \{ m, \ell^2 \}, \ M = \{ m, \ell^3 \}, \ a = \{ \ell^2 \}, \ R = \{ \ell \}, \ \text{and} \ \Delta \bar{\psi} = \{ \ell \} \), with \( n = 6, \ m = 3 \), we expect \( n - m = 3 \) pi groups. They are found as specific using \( \{ 5, 2R \} \) as repeating variables.

\[
\Pi_1 = \int \frac{d^5 R}{\pi^2} \frac{ \bar{z}_3}{z_3} = \left[ \frac{m}{L^3} \right]^2 \left[ \frac{1}{m} \right]^2 \left[ \frac{m}{L} \right] = m^0 \bar{L}^0 \bar{T}^0, \ \text{solve:} \ \alpha = -1.5, \frac{b}{c} = \frac{1}{2}
\]

\[
\Pi_2 = \int \frac{d^5 R}{\pi^2} \bar{z}_3 M^{-1} = \left[ \frac{m}{L^3} \right]^2 \left[ \frac{1}{m} \right]^2 \left[ \frac{L^3}{m} \right]^{-1} = m^0 \bar{L}^0 \bar{T}^0, \ \text{solve:} \ \alpha = 1, \ b = 1, \ c = 2
\]

\[
\Pi_3 = \int \frac{d^5 R}{\pi^2} \bar{z}_3 \bar{z}_3 = \left[ \frac{m}{L^3} \right]^2 \left[ \frac{1}{m} \right]^2 \left[ \frac{L^3}{m} \right] = m^0 \bar{L}^0, \ \text{solve:} \ \alpha = 0, \ b = 0, \ c = 0
\]

The first dimensionless function has its form:

\[
\Pi_1 = \Pi_2 \Pi_3 \quad \text{or,} \quad \frac{\Pi_2 \Pi_3}{\Pi_{\text{MAX}}} = \neq \left( \frac{5 \bar{R}^2}{M}, \ \frac{\bar{R}}{R} \right) \quad (\text{Av1})
\]
For steady flow, we have:

\[ Q_1 + Q_2 + Q_3 = Q_4 \quad \text{or} \quad \frac{1}{2} A_1 + \frac{1}{2} A_2 + \frac{1}{2} A_3 = \frac{1}{2} A_4 \quad \text{(1)} \]

Since \[ 0.2Q_3 = 0.1Q_4 \quad \text{and} \quad Q_4 = 120 \text{ m}^3/\text{h} \],

\[ \frac{Q_4}{2600} = 0.0233 \text{ m}^3/\text{s} \]

\[ \frac{v_3}{2A_3} = \frac{0.0333 \text{ m}^3/\text{s}}{\pi (0.06)^2} = 5.89 \text{ m/s} \quad \text{(Ans3)} \]

Substituting into (1):

\[ v_1 (\pi \frac{D_1}{2})^2 + 5(\pi \frac{D_2}{4})^2 + 5.89 (\pi \frac{D_3}{4})^2 = 0.0333 \]

\[ \Rightarrow v_1 = 5.45 \text{ m/s} \quad \text{(Ans (4))} \]

From mass conservation, \[ Q_4 = \frac{v_4 A_4}{\pi (D_4/2)^2} \]

\[ \frac{(0.0333 \text{ m}^3/\text{s}^2) \times \pi (D_4/2)^2}{v_4 (\pi (D_4/2)^2)} = 5.24 \text{ m/s} \quad \text{(Ans (C))} \]
Q6. The CV (control volume) should surround the tank and wheel and cut through its cable and its exit water jet. Then its horizontal force balance is

\[ F_{nx} = F_{cable} = m_{out} V_{out} = (34 V_j) V_j \cos \theta \]

\[ = 1000 \left( \frac{\pi}{4} \right) (0.04)^2 8^2 \cos 60^\circ \approx 40 \text{ N} \]  

Think: what will happen when \( \theta = 0^\circ \).
07. The control volume surrounds the tank and wheels, and cuts through the jet as shown. We have to assume that splashing into the tank does not increase the x-momentum of the water in the tank. Then we can write the CV horizontal force relation:

\[
\sum F_x = -\int \left( \frac{d}{dt} (up \, du) \right)_{\text{tank}} = -m_{\text{in}} \, u_{\text{in}} = 0 - m_j \, v_j \cos \theta \\
(\text{independence of } \theta)
\]

Thus,

\[
f = (5.9, v_j) \, v_j = 1000 \times \frac{15}{4} \times \left( \frac{5.9}{10} \right) \times 15 \approx 441 \text{ N} \quad (A23)
\]
Q. 8. The horizontal component is

\[ F_H = \frac{F_g \cdot L_g}{A_{vert}} \]

\[ = 1000 \times 9.8 \times (5+1) \times (2 \times 6) \]

\[ \approx 70500 \text{ N} \quad \text{Ans. (a)} \]

The vertical component is the weight of fluid above the quarter-circle panel

\[ F_v = \text{Weight of parallelepiped} - \text{Weight of quarter circle} \]

\[ = \frac{Mg}{\text{box}} - \frac{Mg}{\frac{1}{4} \text{circle}} \]

\[ = 1000 \times (2 \times 7 \times 6) \times 9.8 - 1000 \times 9.8 \times \frac{2}{4} \times 2 \times 6 \]

\[ \approx 638000 \text{ N} \quad \text{Ans. (b)} \]
The scale force is $2 \times 7.81 = 19.62 \text{ N}$. The specific weight $g$ of ethanol is $7733 \text{ N/m}^3$. Then, $f = 19.62 = (\text{Weight} - \text{Buoyancy force}) \text{ case}$

$$= (\gamma_{\text{cube}} - 7733) (0.12)^3$$

Solve for $\gamma_{\text{cube}}$.

$$\gamma_{\text{cube}} = 7733 + \frac{19.62}{0.12^3} \approx 19100 \text{ N/m}^3$$

Answer.
The acceleration sets up pressure 1586 bars which slant down and to the right, in both water and helium. This means there will be a buoyancy force on the balloon up and to its right as shown.

It must be balanced by the string tension down and to its left. If we neglect the balloon material weight, the balloon heads up and to its right at angle

\[ \theta = \tan^{-1} \left( \frac{5}{9.8} \right) \approx 27^\circ (\text{deg}) \]

measured from the vertical. This acceleration—buoyancy effects may seem counterintuitive.
\[ J = 1.255 \text{ kgm}^2, \ \mu = 1.98 \times 10^{-5} \text{ kg/m-s}, \ \beta_D = 0.47 \text{ (ball)}, \ \beta_{D2} = 1.2 \text{ (rod)} \]

\[ \text{Power} = \text{Force} \times \text{velocity} \]

In this case, \( P = 2F_bV_b + 2F_{rod}V_{rod} \)

\[ \omega_0 = \frac{400 \text{ rpm} \times 2\pi}{60} = 41.9 \text{ rad/s} \]

Each ball moves at a centerline velocity, \( V_b = \omega_0 \beta_b = 41.9 \times 0.28 = 11.7 \text{ (0.28 + 0.735/2)} = 13.3 \text{ m/s} \)

Drag force on each ball, \( F_b = \beta_D \frac{1}{2} A \frac{V_b^2}{\beta_{rod}} = 0.47 \left( \frac{1.225}{2} \right) \frac{\pi \times 0.0735^2}{2} \]

\[ R_{rod} \]

\[ V_\beta = \omega_0 \beta_{rod} = 41.9 \times 0.14 = 5.86 \text{ m/s} \]

\[ F_{rod} = \beta_D \frac{1}{2} A \frac{V_\beta^2}{\beta_{rod}} = 1.2 \left( \frac{1.225}{2} \right) (5.86)^2 \times 0.0735 \times 0.28 = 0.0475 \text{ N} \]

\[ P = 2F_bV_b + 2F_{rod}V_{rod} = 4.3 \text{ W} \text{ (Am)} \]